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LAPLACE TRANSFORM AND ITS APPLICATION

A Dissertation submitted for the fulfillment of the M.Sc. Mathematics Semester – IV

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The French Newton
Pierre-Simon Laplace

- Developed mathematics in astronomy, physics, and statistics
- Began work in calculus which led to the Laplace Transform
- Focused later on celestial mechanics

Definition

- The Laplace transform is a linear operator that switched a function f(t) to F(s).
- Specifically: $$F(s) = \mathcal{L}\{f(t)\} = \int_{0^-}^{\infty} e^{-st} f(t)\, dt.$$
  where: $s = \sigma + i\omega.$
- Go from time argument with real input to a complex angular frequency input which is complex.

Restrictions

- There are two governing factors that determine whether Laplace transforms can be used:
  - f(t) must be at least piecewise continuous for $t \geq 0$
  - $|f(t)| \leq Me^{\gamma t}$ where M and γ are constants

ABSTRACT:

- Laplace transform is an extremely potent mathematical tool used in many branches of research and engineering due to their features to solve complex problems in very efficient manner.

- Laplace transforms, like applications of transfer, assist in tackling complicated problems using a fairly simple method as engineering problems become more complex due to their complex structures and thus Laplace transform provides various method to solve problem analytically and so that response of the system is find out.

- Laplace transform has various applications in area of physics, in electric circuit theory, in power system load frequency control and in many more fields which are discussed in this report.

- In this report various examples to illustrate the usefulness of this technique in solving ODE's. Laplace transform is broadly used to solve linear differential equations and particularly initial value problems. Laplace transform reduces an ordinary linear differential equation to an algebraic equation.

INTRODUCTION–

- An integral transform technique called the Laplace transform is very effective for resolving linear ordinary differential equations. It has numerous uses in the fields of optics, mathematics, electrical engineering, control engineering, physics, and signal processing.

- The algebraic operations in the complex plane can be used in place of procedures like differentiation and integration. Thus, one can convert the linear differential equation into algebraic functions of complex variables.

- For determining the ODE-based system's differential equation solution is challenging, so we take into account the system defined by the Transfer Function. On both sides, apply the Laplace transform the differential equation in question. As a result, the differential will change formula into an algebraic formula.

- In order for any function of time f(t) to be Laplace transformable, it must satisfy the following Dirichlet conditions :-

1. The function f (t) must be piecewise continuous which means that it must be single Valued but can have a finite number of finite isolated discontinuities for $t > 0$.

2. The function f (t) must be exponential order which means that f (t) must remain less than se^(−at) as t approaches ∞ where s is a positive constant and a is a real positive

MATERIAL AND METHODS –

- **Definition and Fundamental Properties of the Laplace Transform.**

The Laplace transform is considered as an extension of the idea of the indefinite integral transform: $I\{f(t)\} = \int_0^\infty f(t)dt$

It is defined as follows

Definition –

The **Laplace transform** of f(t), provided it exists, denoted by $\mathcal{L}\{f(t)\}$ is defined by

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

Where s is a real number called a **parameter** of the transform.

Remarks

(a) Laplace transform takes a function f(t) into a function F(s) of the parameter s.

(b) We represent functions of t by lower case letters f, g, and h, while their respective Laplace transforms by the corresponding capital letter F,G, and H. Thus we write

$\mathcal{L}\{f(t)\} = F(s)$ or

$$F(s) = \int_0^\infty e^{-st} ft(t)dt$$

Applications of Laplace Transformations—

- This section describes the applications of Laplace transforms in the areas of science and engineering and due to its ability to solve complex problems in simple way it has very wide scope and numerous applications.

- At first basic application to solve differential , integral and ordinary differential equations in the field of mathematics are described and then application in the area of Physics and Electric Circuit theory is presented which will be followed by a more complex application to power system which includes the description of Load Frequency Control (LFC) for transient stability studies.

1. Applications to Differential and Integral differential equations -

In this section we discuss applications of the Laplace transform and related methods in finding solutions of differential equations with initial conditions and integral equations.

The Laplace transform will allow us to transform an initial-value problem for a linear ordinary differential equation with constant coefficients into a linear algebraic equation that can be easily solved.

(1) Start with differential equation with initial conditions:

$$y'' + y = e^t$$

(2) Take Laplace transform and obtain subsidiary equation:

$$(s^2 + 1)Y(s) = \frac{1}{(s-1)}$$

(3) Solve subsidiary equation for Y:

$$Y(s) = \left(\frac{1}{2(s-1)}\right) + \left(-\frac{1}{2(s^2-1)}\right) + \left(-\frac{1}{2(s^2-1)}\right)$$

(4) Use inverse Laplace transform

to get back to solution:

$$y(t) = \left(\tfrac{1}{2}\right)e^t - \left(\tfrac{1}{2}\right)\cos(t) - \left(\tfrac{1}{2}\right)\sin(t)$$

General Procedure of the Laplace method for solving initial value problems:

Essentially Laplace transform converts initial value problem to an algebraic problem, incorporating initial conditions into the algebraic manipulations. There are three basic steps:

(i) Take the Laplace transform of both sides of the given differential equation, making use of the linearity property of the transform.

(ii) Solve the transformed equation for the Laplace transform of the solution function.

(iii) Find the inverse transform of the expression F(s) found in step (ii).

2. Application of Laplace transform in solving Solution of Systems in analysis of electrical and mechanical systems –

The analysis of mechanical and electrical systems having several components can lead to systems of differential equations that can be solved using the Laplace transform.

Example 4.

Consider the system of differential equations and initial conditions for the functions x and y:

$$x'' - 2x' + 3y' + 2y = 4.$$

$$2y' - x' + 3y = 0.$$

$x(0) = x'(0)=y(0)=0.$ Solve for x and y.

By applying the Laplace transform to the differential equations, incorporating the initial

$$s^2X(s) - 2sX(s)+3sY(s)+2Y(s) = \frac{4}{s}$$

$$2sY(s)-X(s)+3Y(s) = 0.$$

Solve these equations for X(s) and Y(s) to get

$$X(s) = \frac{4s+6}{s^2(s+2)(s-1)} \text{ and } Y(s) = \frac{2}{s\ (s+2)(s-1)}$$

A partial fractions decomposition yields

$$X(s) = -\frac{7}{2}\frac{1}{s} - 3\frac{1}{s^2} + \frac{1}{6}\frac{1}{s+2} + \frac{10}{3}\frac{1}{s-1}$$

and

$$Y(s) = -\frac{1}{s} + \frac{1}{3}\frac{1}{s+2} + \frac{2}{3}\frac{1}{s-1}.$$

Applying the inverse Laplace transform, we obtain the solution

$$x(t) = -\frac{7}{2} - 3t + \frac{1}{6}e^{-2t} + \frac{10}{3}e^{t}$$

and

$$y(t) = -1 + \frac{1}{3}e^{-2t} + \frac{2}{3}e^{t}.$$

3. Application of Laplace transform in area of physics by solving mass damper solutions –

Consider the spring/mass system and Let $x_1 = x_2 = 0$ at the equilibrium position, where the weights are at rest. Choose the direction to the right as positive and suppose the weights are at positions $x_1(t)$ and $x_2(t)$ at time t.

By two applications of Hooke's law, the restoring force on m_1 is

$$-k_1x_1+k_2(x_2-x_1)$$

and that on m_2 is

$$-k_2(x_2-x_1)-k_3x_2.$$

By Newton's second law of motion,

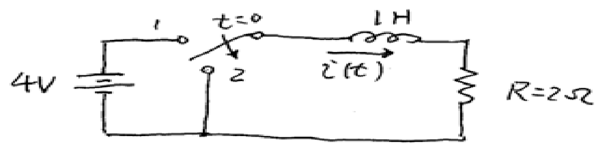
$$m_1x''_1= - (k_1+k_2)x_1+k_2x_2+f_1(t).$$

$$m_2x''_2= k_2x_1-(k_2+k_3)\ x_2 + f_2(t)$$

These equations assume that damping is negligible but allow for forcing functions acting f_1 (t) and f_2 (t) on each mass.

4. Application of Laplace transform in area of Electrical Engineering closed circuit problems –

Example 1.



We have to find $i(t)$ using Laplace transform method for $t>0$

Solution:

(1) Before switched from 1 to 2 at $t=0$

$$i=\frac{4}{2}=2A \Longrightarrow i(0^-)=2A$$

(2) System equation $(t>0)$

KVL:

$$L\frac{di(t)}{dt}+Ri(t)=0 \qquad (L=1H) \quad (R=2ohm)$$

$$\Longrightarrow \frac{di(t)}{dt}+2i(t)=0$$

(3) Solve system equation using Laplace transform

$$L\left[\frac{di(t)}{dt}+2i(t)\right]=L\left[\frac{di(t)}{dt}\right]+2L[i(t)]$$

$$=sI(s)-i(0^-)+2I(s)$$

$$=(s+2)I(s)-2=0$$

$$\Longrightarrow I(s)=\frac{2}{s+2}$$

$$\Longrightarrow i(t)=2e^{-2t}u(t) \ \ A$$

Summary and Future Scopes –

- The Laplace transform is powerful tool using in different areas of mathematics, Physics and engineering for solving complex problem. It provides analysis in both time and frequency domain.

- As Laplace transform provides various properties and simplicity and so that with the ease of application of Laplace transforms in many applications, various research software has made it possible to simulate the Laplace transformable equations directly which has made a good advancement in the research field.

- The Laplace transformation method has been successfully applied to find an exact solution of fractional ordinary differential equations, with constant and variable coefficients and also useful in the field of mathematics, physics and electrical circuit and has a wide future scope.

CONCLUSION –

- This report work presented the various important application of Laplace transform in different areas of physics and electrical power engineering. The main benefits of Laplace transformation are to provide a method to simplify complex problems so that we get required solution and to achieve stability and control.

- Differential equations plays major role in applications of sciences and engineering. It arises in wide variety of engineering applications for e.g. electromagnetic theory, signal processing, computational fluid dynamics, etc.and these equations can be typically solved using either analytical or numerical methods.

- Since many of the differential equations arising in real life application cannot be solved analytically or we can say that their analytical solution does not exist and for such type of problems certain numerical methods exists in the method of Laplace transformation which we discussed above.

Real-Life Applications

- Semiconductor mobility
- Call completion in wireless networks
- Vehicle vibrations on compressed rails
- Behavior of magnetic and electric fields above the atmosphere

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‘Thank You”